

A Test of SU(15) at HERA Using The HELAS Program

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A possible SU(15) process at HERA is investigated. The process that we consider is $e^-P \rightarrow \bar{\nu}_e \mu^- \mu^- + \text{anything}$ through the exchange of new heavy gauge bosons X^- and X^{--} which are predicted in SU(15). This process produces two easily observable like-sign muons in the final state. The cross section of this process is calculated by using HELAS and VEGAS programs, and PDF-library functions. The cross section turns out to be small to be observed in near future.

The standard model, $SU(3) \otimes SU(2) \otimes U(1)$ for strong and electroweak interactions, has been remarkably successful in particle physics. In this model, however, there are too many coupling constants. A straightforward extension of this model is SU(5). The critical drawback of this model is, however, the nonobservation of the proton decay predicted in the model. There have been many grand unification models including SU(15).¹ We have no profound reason to prefer one model to others.

SU(15) is a nonrenormalizable theory since there is no anomaly cancellation in itself. One way to avoid the nonrenormalizability is to introduce mirror fermion. Such introduction of mirror fermion is not favorable but not excluded.¹ Therefore, it is worth reviewing the possible processes arising from SU(15).

In this letter, we investigate the possibility to discover the like-sign dilepton production, $e^-P \rightarrow \bar{\nu}_e l^- l^- + \text{anything}$, in SU(15) in the future e^-P collider. The dileptons are produced via the exchange of new gauge bosons and they can be electrons, muons, or taus. Here, we consider two muons to compare our result to that of Agrawal *et al.*²

We used the HELAS program³ in the numerical calculation of the cross section of the process and the VEGAS program⁴ for the multi-dimensional (10-dimensional in this case) integrations stemmed from the phase spaces of the kinematic variables of the particles, $\bar{\nu}_e, \mu^-$'s and the EHLQ2 nucleon structure function set⁵ for the d quark structure function. The HELAS program is designed mainly to provide numerical means for the calculation of cross sections in the standard model. However, it is easy to implement new interactions from extended standard models like SU(15).

The process that we considered is $e^-P \rightarrow \bar{\nu}_e \mu^- \mu^- + \text{anything}$ through the exchange of heavy gauge bosons X^- or X^{--} . We make use of the dilepton interaction Lagrangian given in the paper of Frampton *et al.*⁶

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$$\begin{aligned}\mathcal{L}_X = & \frac{g_{3l}}{\sqrt{2}} X_\mu^{++} e^T C \gamma^\mu \gamma_5 e - \frac{g_{3l}}{\sqrt{2}} X_\mu^{--} \bar{e} \gamma^\mu \gamma_5 C \bar{e}^T \\ & + \frac{g_{3l}}{\sqrt{2}} X_\mu^+ e^T C \gamma^\mu \left(\frac{1-\gamma_5}{2} \right) \nu_e + \frac{g_{3l}}{\sqrt{2}} X_\mu^- \bar{\nu}_e \gamma^\mu \left(\frac{1-\gamma_5}{2} \right) C \bar{e}^T.\end{aligned}\quad (1)$$

Where, $X^-(X^+)$ and $X^{--}(X^{++})$ are the massive lepton gauge bosons which carry two lepton numbers and result from the $SU(3)_l$ breaking. The coupling constant g_{3l} is given approximately by $1.19e$. Notice that vector couplings are missing for the currents with the doubly charged gauge bosons due to Fermi statistics.

For the process, $e^-P \rightarrow \bar{\nu}_e \mu^- \mu^- + \text{anything}$, in e^-P collider, we are interested in the subprocess, $ed \rightarrow \bar{\nu}_e \mu^- \mu^- u$, through the exchange of gauge bosons, X 's. Where, d quark in the process comes as a parton inside the proton. This process produces two muons and an electron-neutrino, $\bar{\nu}_e$, in the final state while the only incoming lepton is the electron. Therefore, although the total lepton numbers are conserved, lepton numbers in each generation are violated. Since the standard model is free from this violation, this process produces a clean signal without such background processes in the standard model. The relevant Feynman graphs for the process are shown in figs. 1-4.

The new gauge bosons can be produced from the incoming lepton. Then, they produce two muons directly (as shown in fig. 4) or interact with W boson to produce two muons (as shown in figs. 2 and 3) or interact with W boson to produce another doubly charged gauge boson which then turns into two muons (as shown in fig. 1). The corresponding amplitudes are obtained from the Lagrangian given in eq. (1),

$$\begin{aligned}M(1) &= \bar{u}_u \left[-\frac{ig}{\sqrt{2}} \gamma_\mu \left(\frac{1-\gamma_5}{2} \right) U_{ud} \right] u_d \left[\frac{-i(g^{\mu\nu_1} - k_W^\mu k_W^{\nu_1}/M_W^2)}{k_W^2 - M_W^2} \right] \\ &\quad \left(-\frac{ig_{3l}}{\sqrt{2}} \right) \bar{u}_{\mu_1} \gamma_\rho \gamma_5 C \bar{u}_{\mu_2}^T \left[\frac{-i(g^{\rho\nu_3} - k_{X--}^\rho k_{X--}^{\nu_3}/M_{X--}^2)}{k_{X--}^2 - M_{X--}^2} \right] \\ &\quad \left(-\frac{ig}{\sqrt{2}} \right) \left[(k_{X-} + k_{X--})^{\nu_1} g^{\nu_2\nu_3} - k_{X-}^{\nu_3} g^{\nu_1\nu_2} - k_{X--}^{\nu_2} g^{\nu_3\nu_1} \right] \\ &\quad \left(-\frac{ig_{3l}}{\sqrt{2}} \right) v_{\nu_e}^T C \left(\frac{1-\gamma_5}{2} \right) \gamma_\sigma u_e \left[\frac{-i(g^{\nu_2\sigma} - k_{X-}^{\nu_2} k_{X-}^\sigma/M_{X-}^2)}{k_{X-}^2 - M_{X-}^2} \right], \\ M(2) &= \bar{u}_u \left[-\frac{ig}{\sqrt{2}} \gamma_\mu \left(\frac{1-\gamma_5}{2} \right) U_{ud} \right] u_d \left[\frac{-i(g^{\mu\nu} - k_W^\mu k_W^\nu/M_W^2)}{k_W^2 - M_W^2} \right] \\ &\quad \left(-\frac{ig}{\sqrt{2}} \right) \bar{u}_{\mu_1} \gamma_\nu \left(\frac{1-\gamma_5}{2} \right) \left[\frac{i}{\not{k}_{\nu_\mu} - m_{\nu_\mu}} \right] \\ &\quad \left(-\frac{ig_{3l}}{\sqrt{2}} \right) \gamma_\rho \left(\frac{1-\gamma_5}{2} \right) C \bar{u}_{\mu_2}^T \left[\frac{-i(g^{\rho\sigma} - k_{X-}^\rho k_{X-}^\sigma/M_{X-}^2)}{k_{X-}^2 - M_{X-}^2} \right] \\ &\quad \left(-\frac{ig_{3l}}{\sqrt{2}} \right) v_{\nu_e}^T C \left(\frac{1-\gamma_5}{2} \right) \gamma_\sigma u_e, \\ M(3) &= (\mu_1 \leftrightarrow \mu_2),\end{aligned}$$

$$\begin{aligned}
M(4) = & \bar{u}_u \left[-\frac{ig}{\sqrt{2}} \gamma_\mu \left(\frac{1-\gamma_5}{2} \right) U_{ud} \right] u_d \left[\frac{-i(g^{\mu\nu} - k_W^\mu k_W^\nu / M_W^2)}{k_W^2 - M_W^2} \right] \\
& \left(-\frac{ig_{3l}}{\sqrt{2}} \right) \bar{u}_{\mu_1} \gamma_\rho \gamma_5 C \bar{u}_{\mu_2}^T \left[\frac{-i(g^{\rho\sigma} - k_{X--}^\rho k_{X--}^\sigma / M_{X--}^2)}{k_{X--}^2 - M_{X--}^2} \right] \\
& \left(-\frac{ig}{\sqrt{2}} \right) v_{\nu_e}^T C \left(\frac{1-\gamma_5}{2} \right) \gamma_\nu \left(\frac{i}{\not{p}_e - m_e} \right) \left(-\frac{ig_{3l}}{\sqrt{2}} \right) \gamma_\sigma \gamma_5 u_e. \quad (2)
\end{aligned}$$

Where, μ_1 and μ_2 indicate two muons. The vertex, WX^-X^{--} , is similar to triple gauge boson vertices like WWW or ggg in the standard model. U_{ud} is the Cabbibo-Kobayashi-Maskawa (CKM) matrix element for u and d quarks. Here, we have used unitary gauge for the gauge bosons to make use of HELAS subroutines which are written in unitary gauge when the bosons are massive. Only a small modification such as the change in couplings is needed to apply HELAS subroutines to this extended standard model. The total amplitude for the process $e^-d \rightarrow \bar{\nu}_e \mu^- \mu^- u$ is the sum of all the amplitudes in eq. (2),

$$M = M(1) + M(2) + M(3) + M(4). \quad (3)$$

We can simplify the differential cross section for the process by writing the four body phase space as a product of a few two body phase spaces.

$$\begin{aligned}
d\hat{\sigma} &= \frac{|\overline{M}|^2}{2(2\pi)^8 \lambda^{1/2}(\hat{s}, m_e, m_d)} d_4(PS), \\
d_4(PS) &= d_2(e + d \rightarrow A + B) dm_A^2 dm_B^2 d_2(A \rightarrow \bar{\nu}_e u) d_2(B \rightarrow \mu^- \mu^-). \quad (4)
\end{aligned}$$

Where, $d\hat{\sigma}$ is the differential cross section for a parton (quark) q from the proton to be scattered by the electron. $\hat{s} = xs$ is the square of the center of mass energy of the electron and the parton from the proton with the total center of mass energy \sqrt{s} of the system of the incoming electron and the incoming proton. x is the standard Bjorken variable and thus the d quark carries the fractional momentum of the proton, x times the proton momentum. The four body phase space $d_4(PS)$ can be obtained from the process $e^-d \rightarrow \bar{\nu}_e \mu^- \mu^- u$ via the process $e^- + d \rightarrow A + B$ and then $A \rightarrow \bar{\nu}_e + u$ and $B \rightarrow \mu^- + \mu^-$. The two body phase spaces are given as

$$\begin{aligned}
d_2(e + d \rightarrow A + B) &= \frac{1}{8\hat{s}} \lambda^{1/2}(\hat{s}, m_e^2, m_d^2) d\Omega_B^*, \\
d_2(A \rightarrow \bar{\nu}_e u) &= \frac{1}{8m_A^2} \lambda^{1/2}(m_A^2, m_u, m_{\nu}) d\Omega_u^*, \\
d_2(B \rightarrow \mu^- \mu^-) &= \frac{1}{8m_B^2} \lambda^{1/2}(m_B^2, m_{\mu^-}^2, m_{\mu^-}^2) d\Omega_\mu^*. \quad (5)
\end{aligned}$$

m_A is the invariant mass of the system of the u quark and the antineutrino and m_B is the invariant mass of the system of two muons. Angular variables are defined in their center of mass frames. $d\Omega_B^*$ is defined in the center of mass frame of A and B . Similarly, $d\Omega_u^*$ is defined in the center of mass frame of u and $\bar{\nu}_e$ and $d\Omega_\mu^*$ in that of two muons, μ^- and μ^- .

We can calculate the total cross section by summing over all the partons and integrating over x .

$$\sigma = \int_0^1 dx \sum_q F_q(x, Q^2) \hat{\sigma}(\sqrt{\hat{s}} = xs, M_X). \quad (6)$$

Here, instead of summing over all flavors, we need only d quark distribution function to produce two like-sign muons and an antineutrino in the final state. Therefore, the summation of the distribution functions over all the flavors can be simply replaced by the d quark distribution function, $\sum_q F_q(x, Q^2) \rightarrow d(x, Q^2)$. We use this d quark structure function from the EHLQ2 nucleon structure function set in the PDF-library routines in the Phenomenology Institute at the University of Wisconsin. Taking different nucleon structure function sets such as EHLQ1 or KMRSB does not make any physically significant difference.

The numerical value of the cross section of the process is obtained by using HELAS and VEGAS programs. The VEGAS adaptive monte carlo integration subroutines were used for the multi-dimensional integrations arising from the kinematics of the multi-particles involved in the process. In the VEGAS program, the 'FUNCTION' subprogram for the cross section has to be defined in connection with the HELAS program. This FUNCTION subprogram represents the cross section and thus the HELAS program plays the central role in FUNCTION.

Before starting HELAS, we have to prepare a few subroutines. For the coupling which involve in the interactions of the process, the relevant coupling subroutines (COUP1X and COUP2X) should be determined. To find the momenta of the particles in the center of mass frame, we use the momentum subroutine (MOM2CX) along with the subroutines (MOMNTX, BOOSTX) related to the boost factors. Now, we draw the Feynman diagrams of this process and then apply HELAS Feynman rules. This is the central part of the application of HELAS. After setting up all these, we are ready to use HELAS and follow the steps below.

First, we find the wave functions (IXXXXX, OXXXXX) of the external particles by identifying flowing in and flowing out particles (all are fermions in this case). Second, we find the vertex subroutines (JIOXXX, FVOXXX, VVVWXX, IOVXXX) for the corresponding vertices in the Feynman diagrams. Third, we calculate the total amplitude with the phase space factors. Fourth, we multiply EHLQ2 u quark structure function (EHLQ2PDF). Agrawal *et al.* used EHLQ1 for the u quark structure function.² To obtain the numerical result for the cross section, we have to integrate over all relevant kinematic variables by running VEGAS with the above FUNCTION subprogram.

In e^-P collider at HERA, electrons of the energy 30GeV collide on protons of the energy 820GeV in producing the center of mass energy, $\sqrt{s} = 314\text{GeV}$. We chose a few values of the mass of the gauge boson, $M_X \approx 100\text{GeV}$, 200GeV , or 300GeV . We also used the constituent quark masses, $m_u = 0.3\text{GeV}$ and $m_d = 0.3\text{GeV}$. All other standard values were used in the numerical calculation such as $M_W = 80.22\text{GeV}$ and $\sin\theta_W = 0.2325$, etc. With all the values considered, we obtained the following results for the cross section (see Table 1).

The cross section of the process is $8 \times 10^{-2}\text{fb}$ for $M_X = 100\text{GeV}$, $3 \times 10^{-4}\text{fb}$ for $M_X = 200\text{GeV}$, and $3.5 \times 10^{-6}\text{fb}$ for $M_X = 300\text{GeV}$. These results show that the cross section decreases very sharply as the gauge boson X increases. Since the cross section is very small, it is quite difficult to observe the process in the foreseeable future. The result is also an order of 2 smaller than the one in the process considered by Agrawal, Frampton, and Ng.² There, they considered the process, $e^-q \rightarrow e^+2\mu^-q$. In the process, there is no quark flavor changes and there is a positron instead of antineutrino in the final state. The behavior of the sharp declination of the cross section as the increment of M_X is similar to both our process and theirs. Although the cross section of our process is much smaller than that of theirs, our process does not contain well measurable particle, e^+ , in the final state with different kinematics. In our process, there is the missing momentum due to neutrinos without electron or positron in the final state. Therefore, we hope to be able to isolate our process from theirs in the collider of next generation. Unfortunately, however, considering the current luminosity $\sim 10^{31} \text{ cm}^{-2}\text{s}^{-1}$ of HERA we

expect that the cross section of this process is too small to be observed in near future.

Frampton and Ng⁶ showed that the lower bound of the gauge boson mass, M_X , is greater than 120GeV from the experimental angular distribution in Bhabha scattering, $e^+e^- \rightarrow e^+e^-$. Carlson and Frampton⁷ concluded $M_X > 230\text{GeV}$ from the precision data of the angular distribution of electrons in polarized muon decay. Since the value of the coupling, $g_{3l} \approx 1.19e$ is not the unavoidable choice, the above mass lower bound is not strict. And thus Agrawal *et al.*² calculated the cross section of the process, $e^-q \rightarrow e^+2\mu^-q$, with the broad range of M_X , $100\text{GeV} < M_X < 1\text{TeV}$. We also calculated the cross section of the process, $e^-P \rightarrow \bar{\nu}_e\mu^-\mu^- + \text{anything}$, starting with $M_X = 100\text{GeV}$ to compare our result to that of Agrawal *et al.*². However, the lower bound of M_X should be well above 100GeV. Another good way to find the mass limit of the dilepton gauge boson may be to look into the precision electroweak data at the Z peak. The loop calculation with these dilepton gauge bosons should modify the radiative correction to Z or W boson propagators. The comparison of the result with the current data may set a better lower limit on M_X . We have to keep in mind that $SU(15)$ is not a renormalizable theory and some modifications have to be made in the application of the HELAS program in this calculation.

Table 1. The cross section for $e^- + d \rightarrow \bar{\nu} + u + \mu^- + \mu^-$ with various value of M_X

$M_X(\text{GeV})$	cross section in (fb)	cross section of AFN ² in (pb)
100	$\sigma \sim 8.0 \times 10^{-2}$	$\sigma \sim 10^{-2}$
200	$\sigma \sim 3.0 \times 10^{-4}$	$\sigma \sim 10^{-4}$
300	$\sigma \sim 3.5 \times 10^{-6}$	

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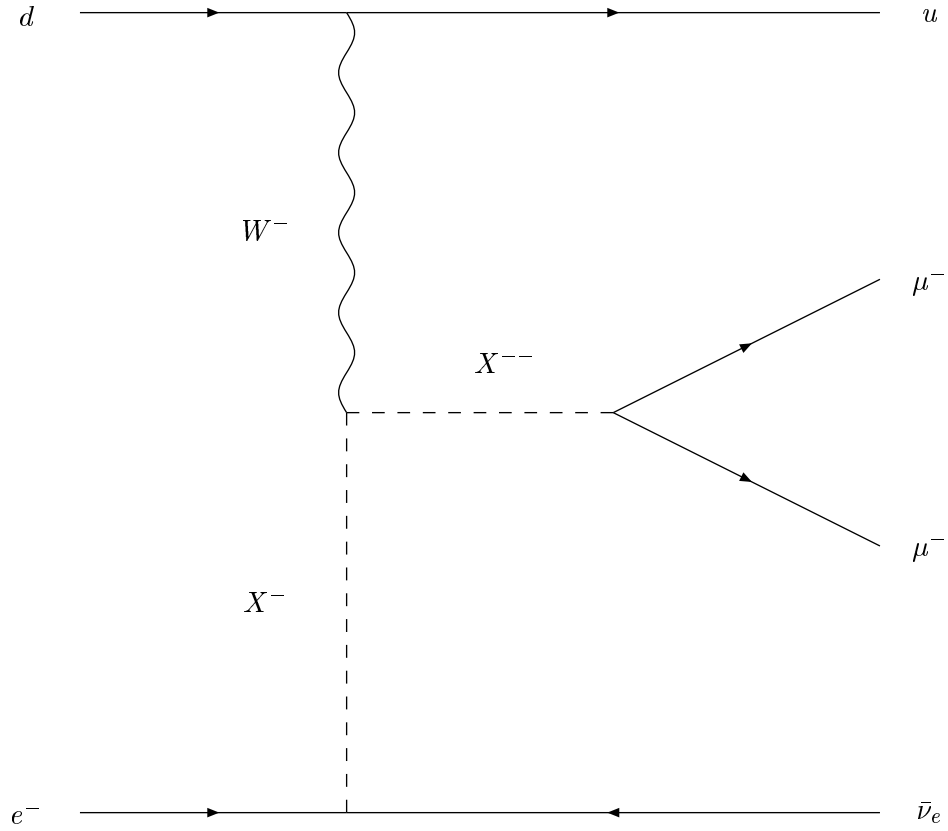


Figure 1. The Feynman diagram for the process $e^- + d \rightarrow \bar{\nu}_e + \mu^- \mu^- + u$ through the X^- , W^- , and X^{--} gauge boson exchanges. This diagram shows that the doubly charged gauge boson X^{--} is produced through W^- and X^- exchanges between e and d and then decays into two muons.

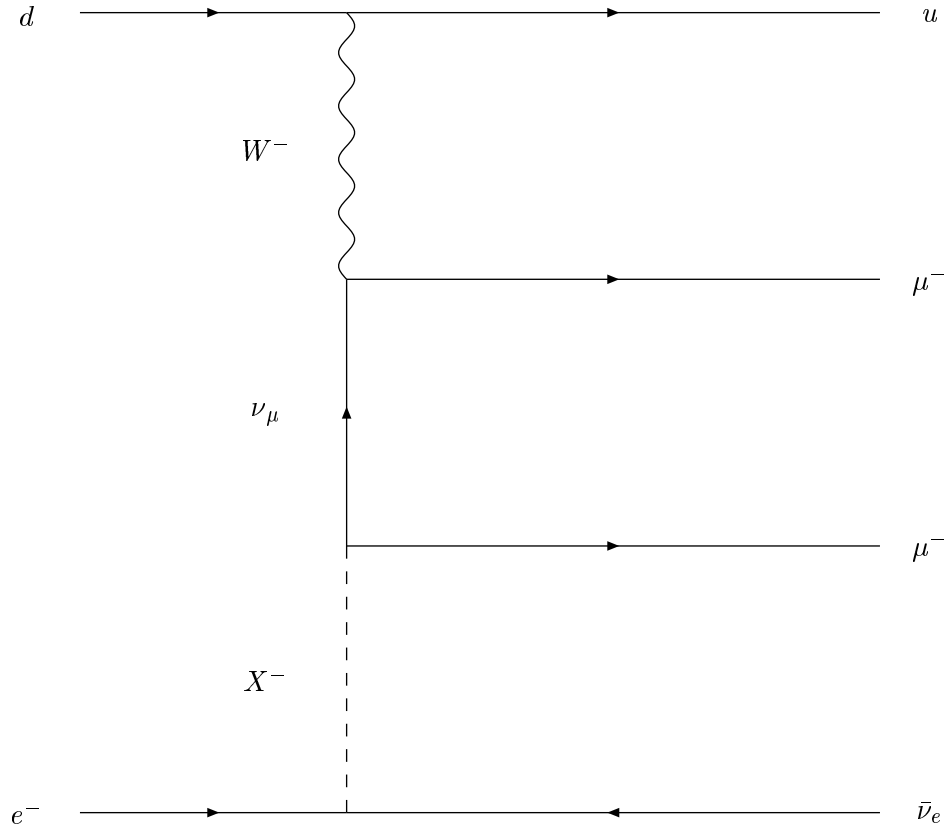


Figure 2. The Feynman diagram for the process $e^- + d \rightarrow \bar{\nu}_e + \mu^- \mu^- + u$ through the X^- gauge boson exchange. This shows that the dileptons are produced through W^- and X^- with the exchange of a neutrino. There are two graphs for this case. This is one of them.

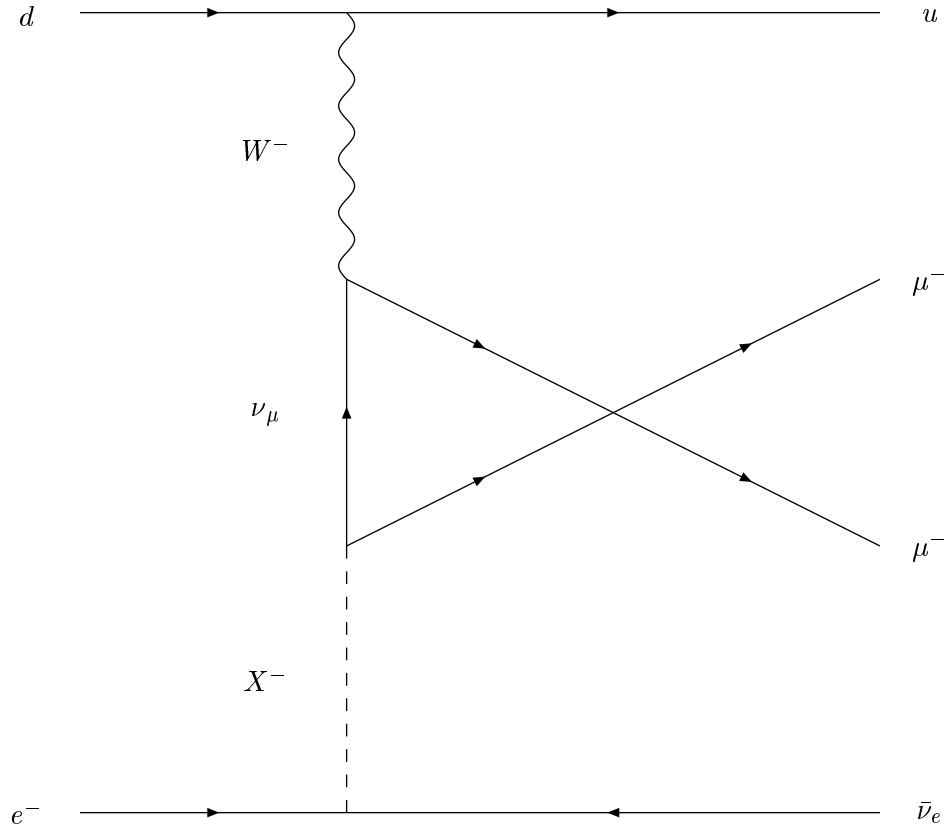


Figure 3. The Feynman diagram for the process $e^- + d \rightarrow \bar{\nu}_e + \mu^- \mu^- + u$ through the X^- gauge boson exchange. This shows that the dileptons are produced through W^- and X^- with the exchange of a neutrino. This is the cross term of the one in fig. 2.

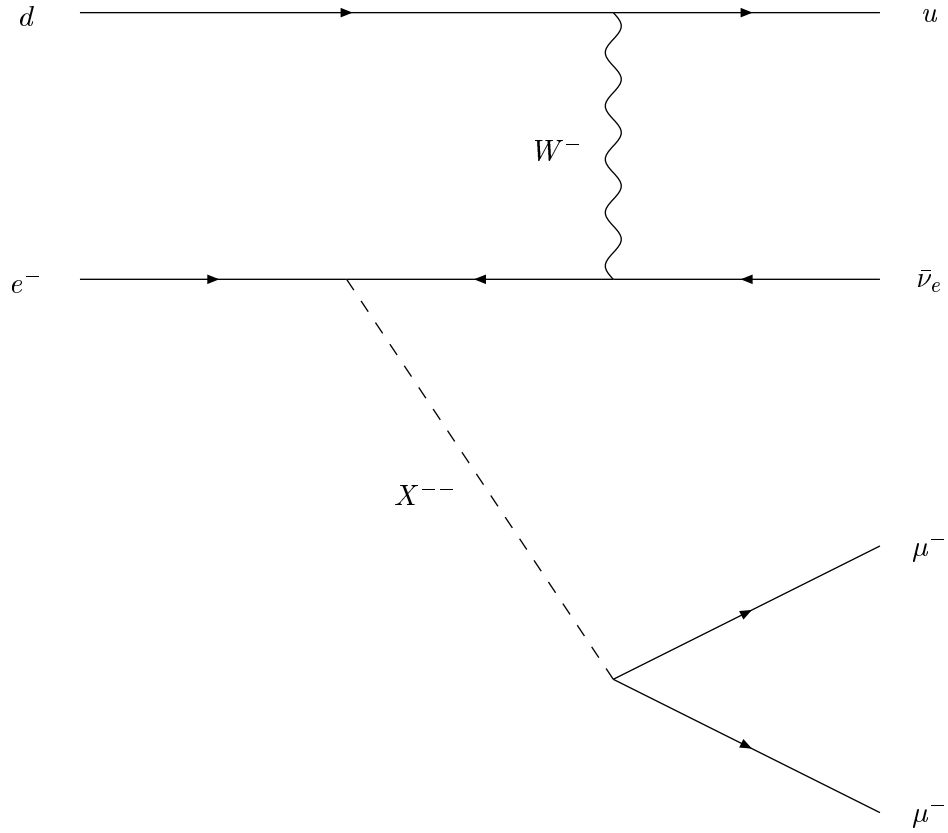


Figure 4. The Feynman diagram for the process $e^- + d \rightarrow \bar{\nu}_e + \mu^- \mu^- + u$ through the X^{--} gauge boson exchange. In this diagram, X^{--} is directly produced from the electron and decays into two muons.

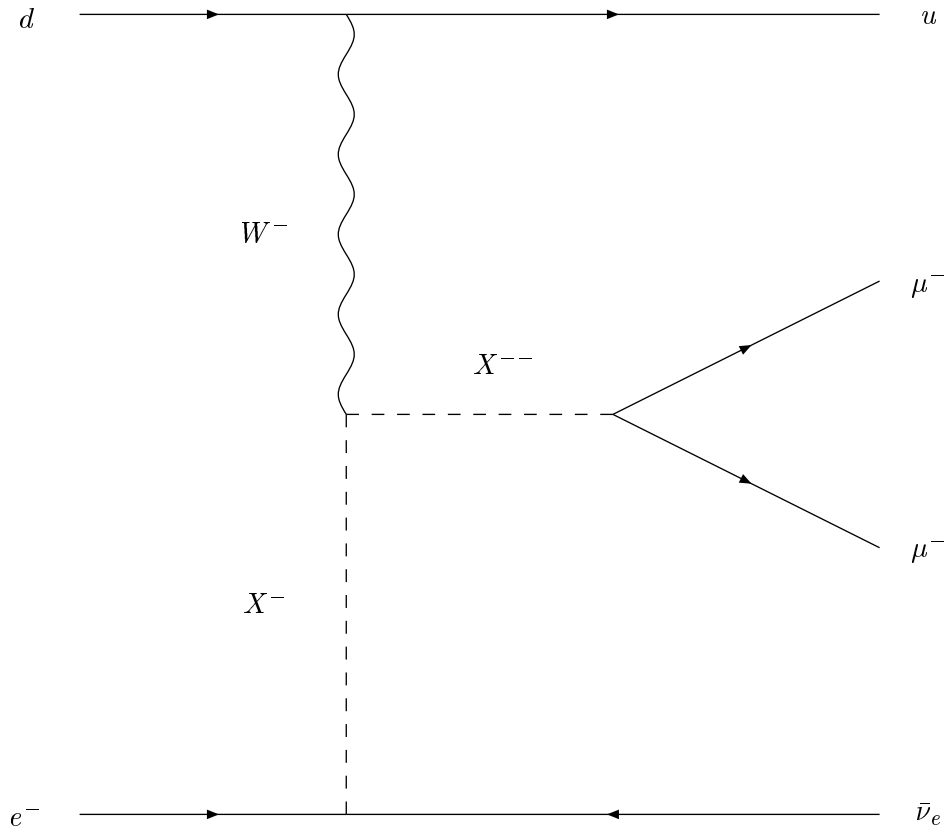


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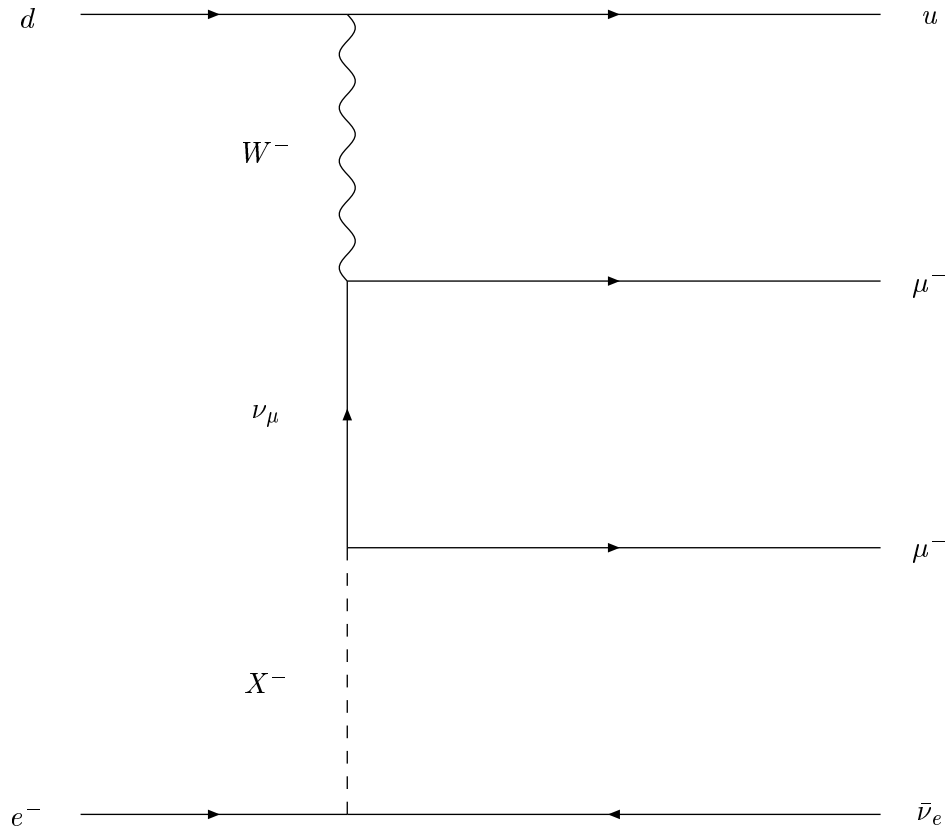


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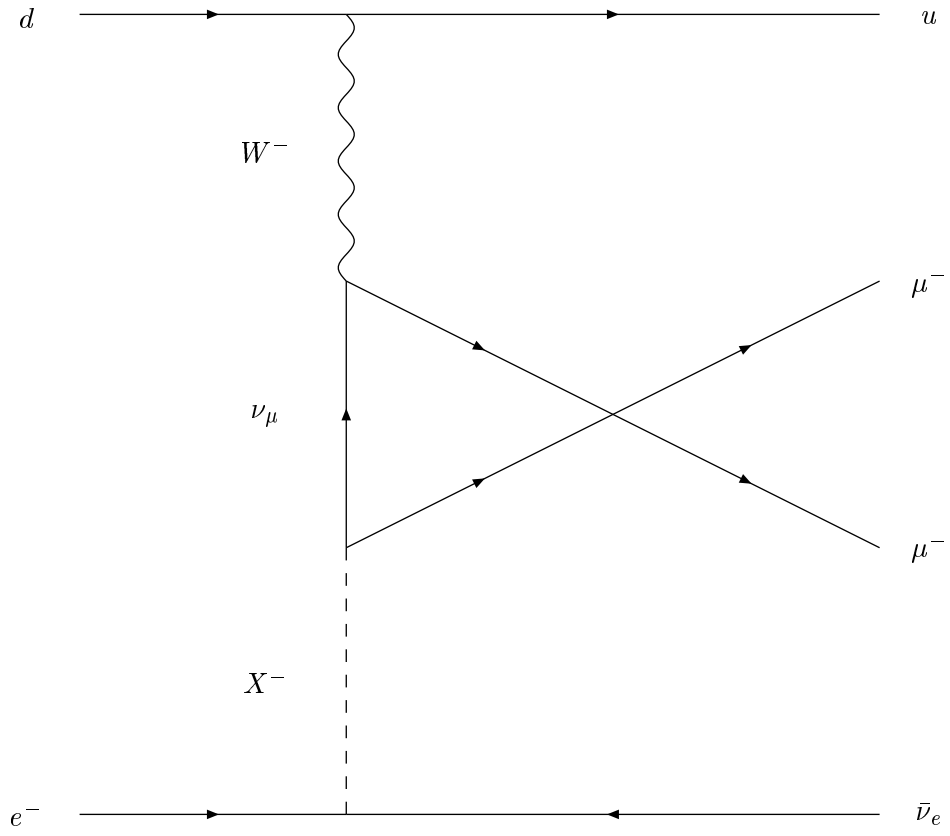


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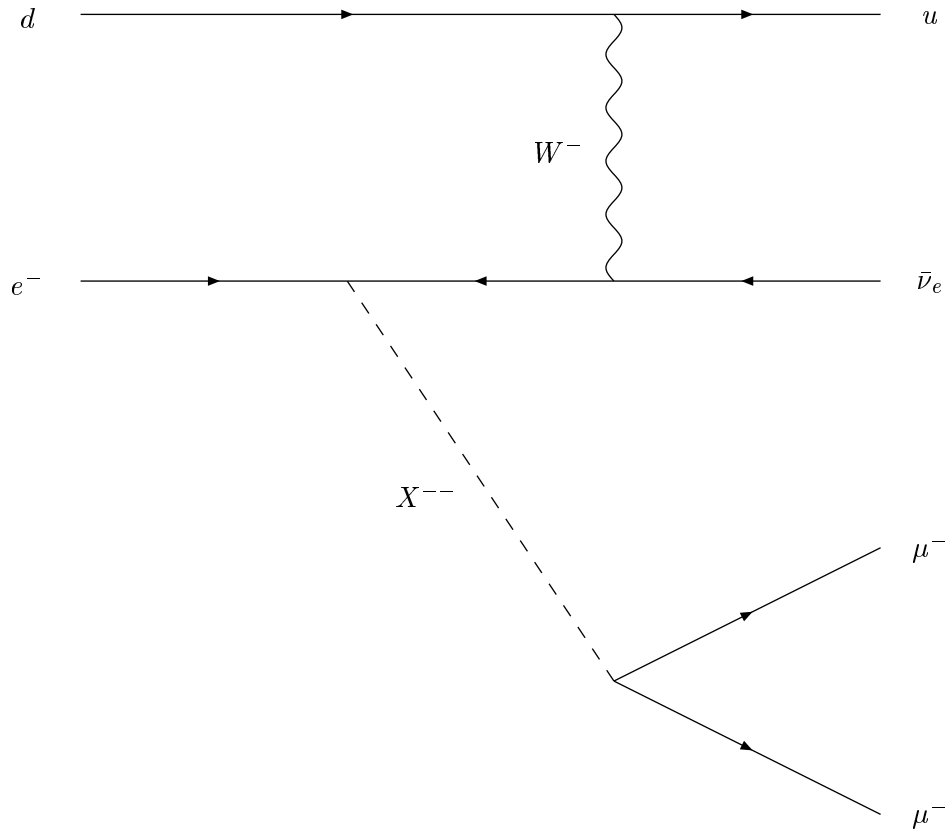


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